Ex1 réducteur

Q1 $T_{\Sigma/R_0} = T_{1/R_0} + T_{2/R_0} + T_{3/R_0}$

* $2T_{1/R_0} = J_1 \omega_1^2$
* $2T_{2/R_0} = J_2 \omega_2^2$
* $2T_{3/R_0} = J_3 \omega_3^2$

or $\frac{\omega_2}{\omega_1} = -\frac{Z_1}{Z_{2a}} \Rightarrow \omega_2 = -\frac{Z_1}{Z_{2a}} \omega_1 = -\lambda \omega_1$

et $\frac{\omega_3}{\omega_1} = \frac{Z_1}{Z_{2a}} \times \frac{Z_{2b}}{Z_3}$ $\omega_3 = \frac{Z_1}{Z_{2a}} \times \frac{Z_{2b}}{Z_3} \omega_1 = \lambda \times \mu \, \omega_1$

Donc $T_{\Sigma/R_0} = \frac{1}{2}\left(J_1 + J_2 \lambda^2 + J_3 \lambda^2 \mu^2\right) \omega_1^2$

Q2 $T_{\Sigma/R_0} = \frac{1}{2} J_{eq} \omega_1^2$ avec $J_{eq} = J_1 + J_2 \lambda^2 + J_3 \mu^2 \lambda^2$

Ex2: système d'entrainement en translation.

Q1 $T_{\Sigma/R_0} = T_{1/R_0} + T_{2/R_0}$.

* $2T_{1/R_0} = J \omega_1^2$
* $2T_{2/R_0} = M V^2$

or $V = R\omega(t)$ (Roulement sans glissement entre le pignon et la cremaillère)

$2T_{2/R_0} = MR^2 \omega^2$

Donc $T_{\Sigma/R_0} = (J + MR^2)\omega^2$

Q2 $T_{\Sigma/R_0} = \frac{1}{2} J_{eq} \cdot \omega^2$ avec $J_{eq} = J + MR^2$

Ex 3 Transformation de mouvement

Q1 $T_{\Sigma/R_0} = T_{1/R_0} + T_{2/R_0}$

$* 2T_{1/R_0} = I\omega^2$

$* 2T_{2/R_0} = mV^2$ avec $V = \frac{p}{2\pi}\omega$

$$\boxed{T_{\Sigma/R_0} = \frac{1}{2}(I\omega^2 + mV^2)}$$

Q2. $2T_{\Sigma/R_0} = I\omega^2 + mV^2$ avec $V^2 = \frac{p^2}{4\pi^2}\omega^2$

$$2T_{\Sigma/R_0} = \left(I + m\frac{p^2}{4\pi^2}\right)\omega^2$$

$$\boxed{J_{eq} = \left(I + \frac{m p^2}{4\pi^2}\right)}$$

Ex 4 Robot cartésien

$\Sigma = \{$astre moteur, réducteur, poulie motrice, poulie receptrice, chariot, solide 2$\}$

Q1 $T_{\Sigma/R_0} = T_{m/R_0} + T_{red/R_0} + T_{p_m/R_0} + T_{p_r/R_0} + T_{ch/R_0} + T_{M2/R_0}$

$* 2T_{m/R_0} = J_m \cdot \omega_m^2$

$* 2T_{red/R_0} = J_r \cdot \omega_m^2$

$* 2T_{p_m/R_0} = J_p \cdot \omega_p^2$

$* 2T_{p_r/R_0} = J_p \cdot \omega_p^2$

$* 2T_{ch/R_0} = M_1 \cdot V_{ch}^2$

$* 2T_{M_2/R_0} = M_2 \cdot V_2^2$

avec $\frac{\omega_p}{\omega_m} = k \iff \omega_p = k\,\omega_m$

$V_{ch} = R_p \cdot \omega_p = R_p \cdot k \cdot \omega_m$

$$T_{\Sigma/R_0} = \frac{1}{2}\left(J_m + J_r + 2J_p k^2 + M_1 R_p^2 k^2\right)\omega_m^2 + \frac{1}{2}M_2 V_2^2$$

Q2 $T_{\Sigma/R_0} = \frac{1}{2}J_{eq}\,\omega_m^2 + B V_2^2$

$$\boxed{\begin{cases} J_{eq} = J_m + J_r + 2J_p k^2 + M_1 R_p^2 k^2 \\ B = \frac{1}{2}M_2 \end{cases}}$$

Ex. 5

Q1 $T_{\Sigma/R_0} = T_{1/R_0} + T_{2/R_0}$

$$* \; 2T_{1/R_0} = {}_A\{\mathcal{C}_{1/0}\} \otimes {}_A\{\mathcal{V}_{1/0}\}$$

$$= {}_A\begin{Bmatrix} m_A \vec{V}_{G_1/R_0} \\ \vec{\sigma}_{A,1/R_0} \end{Bmatrix} \otimes {}_A\begin{Bmatrix} \vec{\Omega}_{1/0} \\ \vec{V}_{A\,1/0} = \vec{0} \end{Bmatrix}$$

$$= \vec{\sigma}_{A,1/R_0} \cdot \vec{\Omega}_{1/0}$$

$$* \; \vec{\sigma}_{A,1/R_0} = [I_{A,1}] \cdot \vec{\Omega}_{1/0} = \begin{bmatrix} A_{1A} & & (0) \\ & B_1 & \\ (0) & & C_{1A} \end{bmatrix}_{R_1} \begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix}_{R_1}$$

$$= C_{1A} \dot\theta \, \vec{z}_{0,1}$$

$$\underline{2T_{1/R_0} = C_{1A} \dot\theta^2}$$

$$* \; 2T_{2/R_0} = {}_{G_2}\{\mathcal{C}_{2/0}\} \otimes {}_{G_2}\{\mathcal{V}_{2/0}\}$$

$$= \begin{Bmatrix} m_2 (\dot\lambda \vec{x}_1 + \lambda \dot\theta \vec{y}_1) \\ \vec{\sigma}_{G_2,2/R_0} \end{Bmatrix} \otimes \begin{Bmatrix} \vec{\Omega}_{2/0} = \dot\theta \vec{z}_{0,1,2} \\ m_2 (\dot\lambda \vec{x}_1 + \lambda \dot\theta \vec{y}_1) \end{Bmatrix}$$

$$* \; \vec{\sigma}_{G_2,2/R_0} = \begin{bmatrix} A_2 & & (0) \\ & B_2 & \\ (0) & & C_{2,G_2} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix} = C_{2,G_2} \dot\theta \, \vec{z}_2 \qquad (\vec{z}_0 = \vec{z}_1 = \vec{z}_2)$$

$$\underline{2T_{2/R_0} = m_2 (\dot\lambda^2 + \lambda^2 \dot\theta^2) + C_{2,G_2} \dot\theta^2}$$

<u>finalement il vient :</u> $T_{\Sigma/R_0} = \frac{1}{2}\left((C_{1,A} + C_{2,G_2} + m_2 \lambda^2) \dot\theta^2 + m_2 \dot\lambda^2 \right)$

Q2 $\underline{A = m_2}$; $\underline{B = C_{1,A} + C_{2,G_2} + m_2 \lambda^2}$

Ex 6 : Rugosimètre.

$$\Sigma = \{1,2,3\} \qquad T_{\Sigma/R_0} = T_{1/R_0} + T_{2/R_0} + T_{3/R_0}.$$

$$\rightarrow 2T_{1/R_0} = {}_A\{C_{1/0}\} \otimes {}_A\{\mathcal{V}_{1/0}\} \qquad {}_A\{C_{1/0}\} = {}_A\left\{\begin{matrix} m_1 \vec{V}_{G_1\in 1/0} \\ \vec{\sigma}_{A,1/R_0} \end{matrix}\right\}$$

$* \ \vec{V}_{G_1 \in 1/0} = \vec{0}$

$* \ \vec{\sigma}_{A,1/R_0}$ (cas particulier A fixe dans R_0)

$$\vec{\sigma}_{A,1/R_0} = \begin{bmatrix} A_1 & & (0) \\ & B_1 & \\ (0) & & J_1 \end{bmatrix}_{B_1} \begin{pmatrix} \dot{\varphi} \\ 0 \\ 0 \end{pmatrix}_{B_1} = J_1 \dot{\varphi} \vec{z}_{1,0}$$

$$\underline{2T_{1/R_0} = \vec{\sigma}_{A,1/R_0} \cdot \vec{\Omega}_{1/0} = J_1 \dot{\varphi}^2}$$

$$\rightarrow 2T_{2/R_0} = {}_B\{C_{2,R_0}\} \otimes {}_B\{\mathcal{V}_{2/0}\}$$

le M^{vt} 2/0 → translation $\vec{x}_0$

$\vec{\Omega}_{2/0} = 0.$

$\vec{V}_{B\in 2/0} = \dot{x}\vec{x}_0$

$$= {}_B\left\{\begin{matrix} m_2 \vec{V}_{G_2/R_0} \\ \vec{\sigma}_{B,2/0} \end{matrix}\right\} \otimes {}_B\left\{\begin{matrix} \vec{0} \\ \dot{x}\vec{x}_0 \end{matrix}\right\}$$

$$\underline{2T_{2/R_0} = m_2 \dot{x}^2}$$

$$\rightarrow 2T_{3/R_0} = {}_{G_3}\{C_{3,0}\} \otimes {}_{G_3}\{\mathcal{V}_{3/0}\}$$

$* \ \underline{\vec{\Omega}_{3/0} = \dot{\theta}\vec{z}_{0,1,2,3}}$

$* \ \vec{V}_{G_3\in 3/0} = \vec{V}_{G_3/0} = \left.\dfrac{d\vec{AG_3}}{dt}\right]_{R_0} = \underline{\dot{x}\vec{x}_0 + r\dot{\theta}\vec{y}_3}$

$$\times \ \vec{\sigma}_{G_3,3/R_0} = \begin{bmatrix} A_3 & -F_3 & -E_3 \\ -F_3 & B_3 & -D_3 \\ -E_3 & -D_3 & C_3 \end{bmatrix}_{G_3} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}_{B_3} = \underline{-E_3\dot{\theta}\vec{x}_3 - D_3\dot{\theta}\vec{y}_3 + C_3\dot{\theta}\vec{z}_3}$$

$$2T_{3/R_0} = \vec{\sigma}_{G_3,3/R_0} \cdot \vec{\Omega}_{3/0} + m_3 \vec{V}_{G_3/R_0} \cdot \vec{V}_{G_3/R_0}$$

$$\underline{= C_3\dot{\theta}^2 + m_3\left(\dot{x}^2 + r^2\dot{\theta}^2 - 2r\dot{x}\dot{\theta}\sin\theta\right)}$$

$$T_{\Sigma/R_0} = \frac{1}{2}\left(J_1\dot{\varphi}^2 + m_2\dot{x}^2 + m_3\dot{x}^2 + m_3 r^2\dot{\theta}^2 - 2m_3 r\dot{x}\dot{\theta}\sin\theta + C_3\dot{\theta}^2\right)$$

Ex 6 : Rugosimètre suite

Q2 $\ _B\{\mathcal{V}_{2/1}\} = \ _B\left\{\begin{matrix}\overrightarrow{\Omega}_{2/1} \\ \overrightarrow{V}_{B\in 2/1}\end{matrix}\right\}$

la liaison entre 2 et 1 est une liaison hélicoïdale par défaut le pas est à droite donc :

$u_{21} = \frac{pas}{2\pi}\omega_{21}$

$$= \ _B\left\{\begin{matrix}\omega_{21} & u_{21} \\ 0 & 0 \\ 0 & 0\end{matrix}\right\}_{(B_0)}$$

Q3. fermeture cinématique de la chaîne $\{0,1,2,0\}$

$$\{\mathcal{V}_{0/0}\} = \vec{0} = \ _B\{\mathcal{V}_{0/2}\} + \ _B\{\mathcal{V}_{2/1}\} + \ _B\{\mathcal{V}_{1/0}\}$$

$$\begin{cases}\overrightarrow{\Omega}_{0/2} + \overrightarrow{\Omega}_{2/1} + \overrightarrow{\Omega}_{1/0} = \vec{0} \\ \overrightarrow{V}_{B\,0/2} + \overrightarrow{V}_{B\,2/1} + \overrightarrow{V}_{B\,1/0} = \vec{0}\end{cases} \qquad \begin{cases}\omega_{21}\vec{x}_0 + \dot{\varphi}\,\vec{x}_0 = \vec{0} \\ -\dot{x}\,\vec{x}_0 + \frac{pas}{2\pi}\omega_{21}\vec{x}_0 = \vec{0}\end{cases}$$

$$\begin{cases}\dot{\varphi} = -\omega_{21} \\ \dot{x} = \frac{pas}{2\pi}\omega_{21}\end{cases} \Rightarrow \dot{x} = -\frac{pas}{2\pi}\dot{\varphi}$$

Q4 $T_{\Sigma/R_0} = \frac{1}{2}\left(J_1 + (m_2+m_3)\frac{pas^2}{4\pi^2}\right)\dot{\varphi}^2 + \frac{1}{2}(C_3 + m_3 r^2)\dot{\theta} + m_3\frac{pas}{2\pi}r\,\dot{\varphi}\,\dot{\theta}\sin\theta$

Donc $J_{eq} = J_1 + (m_2+m_3)\frac{pas^2}{4\pi^2}$

$A = C_3 + m_3 r^2$

$f(t) = m_3 r \frac{pas}{2\pi}\sin\theta$